

Spacetime Quantization from Non-abelian D-particle Dynamics

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ABSTRACT

We describe the short-distance properties of the spacetime of a system of D-particles by viewing their matrix-valued coordinates as coupling constants of a deformed worldsheet σ -model. We show that the Zamolodchikov metric on the associated moduli space naturally encodes properties of the non-abelian dynamics, and from this we derive new spacetime uncertainty relations directly from the quantum string theory. The non-abelian uncertainties exhibit decoherence effects which suggest the interplay of quantum gravity in multiple D-particle dynamics.

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An old idea in string theory is that the classical ideas of general relativity at very small distance scales break down. If one uses only string states as probes of short distance structure, then the usual concept of spacetime ceases to make sense beyond the intrinsic finite length ℓ_s of the string itself [1]. In recent years, however, the discovery of certain solitonic structures, known as D-branes [2], has drastically changed the understanding of the nonperturbative and spacetime properties of string theory. The simplest such string solitons are known as D-particles which are point-like objects to which the endpoints of open strings can attach. It was shown in [3] that, for the non-relativistic scattering of two D-particles with impact parameter of order Δy_i , the space-time uncertainty principle

$$\Delta y_i \Delta t \geq \ell_s^2 \quad (1)$$

yields spatial and temporal indeterminacies $\Delta y_i \geq g_s^{1/3} \ell_s$ and $\Delta t \geq g_s^{-1/3} \ell_s$, where g_s is the string coupling constant and the former bound coincides with the 11-dimensional Planck length ℓ_P which is the characteristic distance scale of M-theory [4]. Thus with weak string interactions, D-particles can probe distances much smaller than ℓ_s .

The target space dynamics of a system of N D-particles is usually described by a matrix quantum mechanics which is obtained by dimensionally reducing ten-dimensional $U(N)$ Yang-Mills theory to the worldlines of the D-particles [5]. The D-particle coordinates are given by $N \times N$ Hermitian matrices Y_{ab}^i which are to be thought of as adjoint Higgs fields. The diagonal components Y_{aa}^i represent the positions of the N D-particles themselves while the off-diagonal components Y_{ab}^i correspond to short oriented open string excitations between D-particles a and b when the two objects are brought infinitesimally close to each other (fig. 1). In this letter we shall adopt a different formalism by treating the Y_{ab}^i as coupling constants of a deformed worldsheet conformal field theory [6, 7], which is the pertinent field theory for the weak-coupling regime of the dynamics. We will show that the small-scale structure of spacetime is naturally encoded within the Zamolodchikov metric [8] on the moduli space \mathcal{M} of σ -model couplings, which itself captures all of the non-trivial dynamical information about the D-particle system. Quantization of \mathcal{M} is achieved by summing over worldsheet genera of the deformed σ -model. This promotes the D-particle couplings to quantum operators in target space from which non-trivial uncertainty relations can be derived. In this way we will find, directly from the quantum string theory itself, new quantum mechanical uncertainties which are particular to the non-abelian nature of D-particle dynamics, in addition to the uncertainties implied by (1). These smearings occur among the open string excitations between different D-particles and they yield a triple space-time uncertainty relation which implies that the scattering of D-particles at high energies can probe very small distances through their string interactions. We will also show that there is a smearing with respect to different *spatial* string coordinates which is rather different in spirit than the short-large distance duality relation (1). This represents a proper quantization of the noncommutative space-time implied by D-particle field theory and thereby yields limitations on the accuracy of

simultaneously locating different spatial directions. These correlations are also given by energy-dependent distances, so that the non-abelian uncertainties exhibit decoherence effects which are characteristic of quantum gravity. This construction therefore illuminates the manner in which D-particle interactions probe very short distances where the effects of quantum gravity are significant.

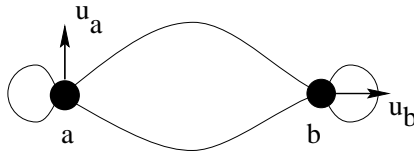


Figure 1: *Emergence of $U(N)$ gauge symmetry for bound states of N D-particles (solid circles). An oriented fundamental string (thin lines) can start and end either at the same or different D-particle, with energy proportional to their separation, giving N^2 massless vector states when the particles are practically on top of each other. These states form a representation of $U(N)$. For the configuration shown, D-particles a and b move at mutually transverse velocities u_a and u_b in target space.*

The tree-level $N \times N$ matrix D-particle dynamics is described by the path integral [2]

$$Z_N[Y] = \langle W[\partial\Sigma; Y] \rangle_0 \equiv \int Dx e^{-S_0[x]} \text{tr} \mathcal{P} \exp \left(\frac{ig_s}{\ell_s^2} \oint_{\partial\Sigma} Y_i(x^0(s)) dx^i(s) \right) \quad (2)$$

where $S_0[x] = (1/2\ell_s^2) \int_{\Sigma} (\partial x^\mu)^2$ is the free worldsheet σ -model action which is defined on a disc Σ whose boundary $\partial\Sigma$ is a circle parametrized by $s \in [0, 1]$. The elementary string fields x^μ are maps from Σ into flat 9+1-dimensional spacetime, and the path-ordered Wilson loop operator in (2) is written in the temporal gauge for the dimensionally-reduced $SU(N)$ Chan-Paton gauge field $A_i = (1/\ell_s^2) Y_i$. To write (2) in the form of a local deformation of the conformal field theory $S_0[x]$, we disentangle the path ordering by introducing one-dimensional complex auxilliary fields $\bar{\xi}_a(s), \xi_b(s)$ on $\partial\Sigma$ [9] and writing

$$W[\partial\Sigma; Y] = \int D\bar{\xi} D\xi \bar{\xi}_c(0) \exp \left[- \int_0^1 ds \bar{\xi}_a(s) \left(\delta^{ab} \partial_s + \frac{ig_s}{\ell_s^2} Y_i^{ab}(x^0(s)) \partial_s x^i(s) \right) \xi_b(s) \right] \xi_c(1) \quad (3)$$

The auxilliary quantum fields have propagator $\langle \bar{\xi}_a(s) \xi_b(s') \rangle = \delta_{ab} \Theta(s' - s)$, where $\Theta(s)$ is the usual step function. The partition function (2) now takes the form of a path integral involving a local action $S_0[x] + \oint_{\partial\Sigma} Y_i^{ab}(x^0(s)) V_{ab}^i(x; s)$ and matrix-valued vertex operators $V_{ab}^i(x; s) = (ig_s/\ell_s^2) \partial_s x^i(s) \bar{\xi}_a(s) \xi_b(s)$.

The emergence of non-trivial spacetime uncertainty relations can be seen most directly by treating the D-particles as heavy non-relativistic objects. Their collective coordinates can then be described by recoil operators [10] which are induced by the scattering of string matter off the D-particle background. The deformation is given by $Y_i^{ab}(x^0) = \ell_s(Y_i^{ab} \ell_s \epsilon + U_i^{ab} x^0) \epsilon \Theta_\epsilon(x^0)$, where the D-particle spatial coordinates are identified with the couplings Y_i and the U_i correspond to their Galilean recoil velocities. The parameter

$\epsilon \rightarrow 0^+$ regulates the ambiguous value of $\Theta(s)$ at $s = 0$, which ensures that the D-particle system starts moving only at time $x^0 = 0$. It is related to the worldsheet ultraviolet cutoff scale Λ (measured in units of the size of Σ) by $\epsilon^{-2} = 2\ell_s^2 \log \Lambda$. For finite ϵ , these operators each have an anomalous dimension $-\frac{1}{2}|\epsilon|^2 < 0$ [10] and thus lead to a relevant deformation of $S_0[x]$. The corresponding β -functions are [7] $dY_i^{ab}/dt = U_i^{ab}$, $dU_i^{ab}/dt = 0$, which are the Galilean evolution equations for the D-particles if we identify the time $t = \ell_s \log \Lambda$ with the worldsheet scale.

The natural geometry on the moduli space \mathcal{M} of deformed conformal field theories described above is given by the Zamolodchikov metric [8] which is the two-point function $G_{ab;cd}^{ij} = 2\Lambda^2 \langle V_{ab}^i(x;0) V_{cd}^j(x;0) \rangle$ in the full model (2). Using the representation (3), after a long perturbative calculation we find [11]

$$G_{ab;cd}^{ij} = \frac{4g_s^2}{\ell_s^2} \left[\delta^{ij} I_N \otimes I_N - \frac{g_s^2}{6} \left\{ I_N \otimes (U^i U^j + U^j U^i) + U^i \otimes U^j + U^j \otimes U^i + (U^i U^j + U^j U^i) \otimes I_N \right\} \right]_{db;ca} + \mathcal{O}(g_s^6) \quad (4)$$

where I_N is the $N \times N$ identity matrix and we have renormalized g_s to the time-independent coupling $g_s/|\epsilon|\ell_s$. The metric (4) is a complicated function of the D-particle dynamical parameters, which will be the key to its use in probing short-distance structure on \mathcal{M} . One can also perturbatively calculate the canonical momentum P_{ab}^i of the D-particle system, which, in the Schrödinger picture, is given by the expectation value of $-i\delta/\delta Y_i^{ab}$ with respect to (2), i.e. $P_{ab}^i = \langle V_{ab}^i(x;0) \rangle$. Another long and tedious calculation gives [11]

$$P_{ab}^i = \frac{8g_s^2}{\ell_s} \left[U^i - \frac{g_s^2}{6} (U_k^2 U^i + U_k U^i U^k + U^i U_k^2) \right]_{ba} + \mathcal{O}(g_s^6) \quad (5)$$

which coincides with the contravariant velocity $P_{ab}^i = \ell_s G_{ab;cd}^{ij} \dot{Y}_j^{cd}$ on \mathcal{M} . The final quantity we need is the Zamolodchikov C -function, which interpolates on \mathcal{M} among two-dimensional field theories according to the C -theorem $\partial \mathcal{C}/\partial t = e^{-Ct} \dot{Y}_i^{ab} G_{ab;cd}^{ij} \dot{Y}_j^{cd}$ [8]. This differential equation can be solved for small velocities to give the physical target space time coordinate [6]

$$T \equiv \sqrt{\ell_s \mathcal{C}(t)} \quad t \simeq 2g_s t \sqrt{\frac{1}{\ell_s} \mathcal{F}(U)} \left(\int_0^t d\tau \, e^{2(\tau^2 - t^2)g_s^2 \mathcal{F}(U)/\ell_s^2} \right)^{1/2} \quad (6)$$

where we have introduced the scalar function

$$\mathcal{F}(U) = \text{tr } U_i^2 - \frac{g_s^2}{3} \text{tr} [2U_i^2 U_j^2 + (U_i U_j)^2] + \mathcal{O}(g_s^4) \quad (7)$$

The moduli space dynamics can be derived from the Lagrangian $-\frac{\ell_s}{2} \dot{Y}_i^{ab} G_{ab;cd}^{ij} \dot{Y}_j^{cd} - \mathcal{C}$ [6] which can be shown [11] to coincide to leading orders with the non-abelian Born-Infeld effective action [12] for the target space D-particle dynamics.

The quantization of \mathcal{M} comes from summing over all worldsheet genera of the σ -model (2). The dominant contributions come from pinched annulus diagrams of pinching size $\delta \rightarrow 0$ [6]. Symbolically, this approximation leads to a genus expansion of the form

$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \quad (8)$$

consisting of thin tubes (worldsheet wormholes) attached to Σ . The attachment of each tube corresponds to inserting a bilocal pair $V_{ab}^i(x; s)V_{cd}^j(x; s')$ on $\partial\Sigma$, with interaction strength g_s^2 , and computing the string propagator along the thin strips. There are modular $\log \delta$ divergences that arise, which should be identified with worldsheet divergences at lower genera [6], and hence we take $\log \delta = 2g_s^\eta \log \Lambda = \frac{1}{\ell_s^2} g_s^\eta \epsilon^{-2}$ (the exponent $\eta \geq 0$ is left arbitrary for the moment in order to compare with other results later on). The effects of the dilute gas of wormholes (8) on Σ are to exponentiate the bilocal operator leading to a change in the action in (2, 3). This contribution can be cast into the form of a local action by rewriting it as a functional Gaussian integral over wormhole parameters ρ_i^{ab} , and we arrive finally at [11]

$$\sum_{\text{genera}} Z_N[Y] \simeq \left\langle \int_{\mathcal{M}} D\rho \, e^{-\rho_i^{ab} G_{ab;cd}^{ij} \rho_j^{cd} / 2|\epsilon|^2 \ell_s^2 g_s^2 \log \delta} W[\partial\Sigma; Y + \rho] \right\rangle_0 \quad (9)$$

From (9) we see that the effect of the resummation over pinched genera is to induce quantum fluctuations of the solitonic background, giving a statistical Gaussian spread to the D-particle couplings. Note that the width of the Gaussian distribution in (9), which we identify as the wavefunction of the system of D-particles [6], is time independent and represents not the spread in time of a wavepacket on \mathcal{M} , but rather the true quantum fluctuations of the D-particle coordinates.

The corresponding spatial uncertainties can be found by diagonalizing the Zamolodchikov metric (4). We apply a Born-Oppenheimer approximation to the D-particle interactions, which is valid for small velocities [3], to separate the diagonal D-particle coordinates from the off-diagonal parts of the Higgs fields representing the short open string excitations connecting them. This is achieved via a time independent gauge transformation $Y^i = \Omega \text{diag}(y_1^i, \dots, y_N^i) \Omega^{-1}$, where $\Omega \in U(N)$ describes the fundamental strings and the eigenvalues $y_a^i \in \mathbb{R}$ represent the positions of the D-particles which move at velocities $u_a^i = \dot{y}_a^i$. This diagonalizes (4) in its colour indices. To diagonalize the resulting tensor in its spacetime indices $i, j = 1, \dots, 9$ we consider the case where, for simplicity, two given D-particles a, b move orthogonally to each other, as depicted in fig. 1, with the same speed $|u_a| = u$. For $a = b$, there is only one non-zero eigenvalue $\lambda_{aa}^1(u) = 6u^2$, owing to the fact that a single D-particle on its own has only open string excitations starting and ending on itself. Upon rotation to the one-dimensional frame spanned by the velocity vector u_a^i/u , the spacetime diagonalizing matrix $O_{aa}(u)$ is the 9×9 identity matrix. We shall refer to this frame as the ‘string frame’, because it represents the coordinate system relative to the open string excitations of the D-particles. For $a \neq b$, there are two non-zero

eigenvalues $\lambda_{ab}^\pm(u) = (2 \pm \sqrt{5})u^2$, and the dimension of the string frame increases by one since in this case the string stretches between two different D-particles. We parametrize the plane spanned by u_a and u_b as $u_a^i = u\delta^{i,1}$ and $u_b^i = u\delta^{i,2}$, and then the orthogonal diagonalizing matrix $O_{ab}(u)$ has a block diagonal form consisting of the 2×2 block matrix $\frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{5} & -1 \\ 1 & \sqrt{5} \end{pmatrix}$ and the 7×7 identity matrix.

The coordinate transformation $\tilde{Y}_{ab}^i = O_{ab}^{ji}[\Omega^{*-1}Y_j\Omega]_{ba} \equiv O_{ab}^{ji}X_{ab}^j$, where the collective D-particle coordinates $X^i(Y)$ contain information about the open string excitations, diagonalizes the bilinear form in (9) and yields the statistical variances $(\Delta\tilde{Y}_{ab}^i)(\Delta\tilde{Y}_{ab}^i)^\dagger = \ell_s^2 g_s^\eta (1 + \frac{g_s^2}{48\pi^3} \lambda_{ab}^i(u)) + \mathcal{O}(g_s^4)$. For $a = b$ we therefore arrive at the uncertainties

$$|\Delta X_{aa}^i| = \ell_s g_s^{\eta/2} \left(1 + \frac{g_s^2}{8\pi^3} u^2 \delta^{i,1} \right) + \mathcal{O}(g_s^4) \geq \ell_s g_s^{\eta/2} \quad (10)$$

for the individual D-particle coordinates. For $\eta = 0$ the minimal length in (10) coincides with the standard string smearing [1] while for $\eta = \frac{2}{3}$ it matches ℓ_P which arises from the kinematical properties of D-particles [3]. A choice of $\eta \neq 0$ is more natural since the modular strip divergences should be small for weakly interacting strings. Note that the uncertainty (10) is always larger in the string frame, representing the additional smearing that occurs from the open string excitations on the D-particles. Outside of this frame we obtain exactly the standard stringy smearings directly from the worldsheet formalism, without the need of postulating an auxilliary uncertainty relation as is done in [1, 3].

The coordinate uncertainties for $a \neq b$ are responsible for the emergence of a true non-commutative quantum spacetime and represent the genuine non-abelian characteristics of the D-particle dynamics. In this case we find the same velocity independent spreads as in (10) for the coordinates in the space orthogonal to the two-dimensional string frame. In the string frame we may assume, by symmetry, that $\Delta X_{ab}^1 \sim \Delta X_{ab}^2$, and then we arrive at a system of two linear equations in two unknowns. Solving them simultaneously gives a minimum length uncertainty ΔX_{ab}^1 of the same form as (10) and the additional correlator

$$\text{Re} \left\langle \left\langle X_{ab}^1 X_{ab}^2 \right\rangle \right\rangle_{\text{conn}}^{(\rho)} = \frac{\ell_s^2}{16\pi^3} g_s^{2+\eta} u^2 \quad , \quad a \neq b \quad (11)$$

where $\langle \langle \cdot \rangle \rangle_{\text{conn}}^{(\rho)}$ denotes the connected statistical correlation function with respect to the wormhole probability distribution in (9). When the D-particles are in motion, we see from (11) that the associated open string position operators are not independent random variables and have a non-trivial quantum mechanical correlation. This is a new form of spacetime quantum uncertainty relations between different *spatial* directions of the target space. For $\eta = \frac{2}{3}$, the right-hand side of (11) can be written in terms of ℓ_P^2 when we identify the kinetic energy of the D-particles with $g_s^2 u^2$, as follows from (5). The energy dependence of (10) and (11) is a quantum decoherence effect which can be understood from a generalization of the Heisenberg microscope whereby we scatter a low-energy probe, represented by a closed string state, off the D-particle configuration (see fig. 1). As the

closed string state hits a D-particle, it splits into two open string states, represented by the recoil of the particle upon impact with the detector, which absorb energy from the scattering. For an isolated D-particle, these open string excitations have their ends attached to the same point. For two D-particles the ends of the open string can attach to different points. Since the recoil of the constituent D-particles causes the bound state configuration to recoil as well, the interactions mediated by the open strings cause a non-trivial correlation between different coordinate degrees of freedom stretched between the two particles. Only when there is no recoil ($u = 0$) can one measure independently the positions of the two D-particles.

We can derive space-time uncertainty principles by exploiting the canonical structure on \mathcal{M} represented by (5). In the present context, whereby quantum mechanical averages are identified with vacuum expectation values of the σ -model on Σ , the uncertainty $(\Delta P_{ab}^i)^2$ in the momentum can be computed as the difference between (4) for $a = c, b = d, i = j$ and the square of (5). For weakly coupled strings this yields $(\Delta P_{ab}^i)^2 = \frac{4g_s^2}{\ell_s^2} \delta_{ab} + \mathcal{O}(g_s^4)$, and using the minimal length uncertainties computed above we find the canonical uncertainty relation $\Delta Y_i^{ab} \Delta P_{cd}^j \geq 2g_s^{1+\eta/2} \delta_i^j \delta_c^a \delta_d^b$. Upon summation over genera the target space time coordinate (6) becomes a quantum operator [13], unlike the situation in conventional quantum mechanics. Within the present Born-Oppenheimer approximation, we can expand the function (7) as a power series in $|U_{ab}|/u \ll 1$, $a \neq b$. Then using the leading Heisenberg commutation relations between Y and $P \sim 8g_s^2 U/\ell_s$, we arrive at the space-time commutation relation

$$[Y_i^{ab}, T] \simeq 2i \ell_s^2 g_s^{\eta/2} \left(\delta^{ab} + (1 - \delta^{ab}) \frac{U_i^{ba}}{2u} \right) \quad (12)$$

to leading order in g_s (or equivalently in the off-diagonal velocity expansion). From (12) we infer the space-time uncertainty relation $\Delta Y_i^{aa} \Delta T \geq \ell_s^2 g_s^{\eta/2}$ for the D-particle coordinates, which for $\eta = 0$ yields the standard lower bound (1) which is independent of g_s , as argued in [3] from a basic string theoretic point of view. But then the minimal distance (10) doesn't probe scales down to ℓ_p . This fact can be understood by noting that T is not the same as the longitudinal worldline coordinate of a D-particle, as is assumed in (1), but is rather a collective time coordinate of the D-particle system which is induced by all of the interactions among the particles. However, we can adjust the uncertainty relations to match the dynamical properties of 11-dimensional supergravity by multiplying the definition (6) by a factor $g_s^{-\eta/2}$, which implies that with weak string interactions the target space propagation time for the D-particles is very long.

To see the effects of the string interactions between D-particles, we again use the canonical smearing between Y_j^{ab} and P_{ab}^i for $a \neq b$ in (12) to arrive at a triple uncertainty relation

$$(\Delta Y_i^{ab})^2 \Delta T \geq \frac{\ell_s^3 g_s^{\eta/2-1}}{16u} \quad , \quad a \neq b \quad (13)$$

The uncertainty principle (13) implies that the system of D-particles, through their open

string interactions, can probe distances much smaller than the characteristic distance scale in (13), which for $\eta = \frac{2}{3}$ is $\ell_P \ell_s^2$, provided that their kinetic energies are large enough. Of course, in the fully relativistic case the existence of a limiting velocity $u < 1$ implies a lower bound on (13). With the minimum spatial extensions obtained above, this bound yields the characteristic temporal length $\Delta T \geq g_s^{-1/3} \ell_s$ for D-particles [3]. Triple uncertainty relations involving only ℓ_P^3 have been suggested in [3] based on the holographic principle of M-theory.

The kinetic energy dependence of (11) and (13) is the main distinguishing feature between D-particle dynamics and ordinary quantum mechanics, since it implies a bound on the measurability of lengths that depends entirely on the energy content of the system. Such uncertainties, where the bound on the accuracy with which one can measure a quantity depends on its size, are characteristic features of decoherence in certain approaches to quantum gravity [13]. In the present case the quantum coordinate fluctuations, due to the open string excitations between D-particles, can lead to quantum decoherence for a low energy observer who cannot detect such recoil fluctuations in the sub-Planckian space-time structure [6]. This approach to short-distance physics using non-abelian D-particle dynamics therefore seems to have naturally encoded within it features of quantum gravity. These foamy properties of the noncommutative structure of the D-particle spacetime might require a reformulation of the phenomenological analyses of length measurements as probes of quantum gravity. More details about these constructions will appear in a forthcoming paper [11].

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